UNM-PNM Statewide High School Mathematics Contest LVIII
Round 2, 14 February 2026, 13:00-16:00

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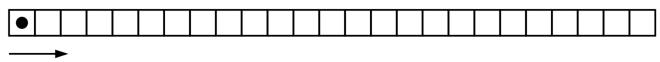
1. (a) Cora and Ernie are playing a new board game. The game board has 25 squares arranged in a 1×25 rectangle. They take turns moving a marker either 1 or 2 spaces forward, with Cora moving first. The marker is initially in a square at the end of the board, as shown, and must always be moved towards the other end. A player who cannot make a move that stays on the board loses. Which of the players can ensure victory?



(b) What if the game board is now 1×2026 and the marker can be moved 1, 2, or 3 spaces. Can one of the players ensure victory?

2. (a) Consider four cats that wish to occupy a condo. There are eight condos in a row. Each cat picks a condo at random to occupy independently of the other cats (and each condo can hold any number of cats). Find the probability that each cat chooses a different condo.

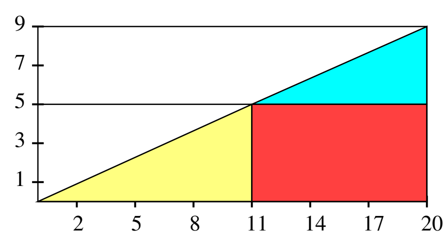
(b) Solve the problem for n cats and $2n$ condos for $n > 4$.

3. Consider the following figure which is drawn accurately to scale.

(a) Show that the right triangle whose vertices are $(0,0)$, $(20,0)$, and $(20,9)$ has area 90.

(b) Show that the triangle whose vertices are $(0,0)$, $(11,0)$, and $(11,5)$ has area 27.5. Show that the rectangle whose vertices are $(11,0)$, $(20,0)$, $(20,5)$, and $(11,5)$ has area 45. Finally, show that the triangle whose vertices are $(11,5)$, $(20,5)$, and $(20,9)$ has area 18. Whence the area of the shaded region is 90.5

(c) Explain why the results in **(a)** and **(b)** differ.



4. (a) How many positive integers strictly less than 2026 are multiples of 3 <u>and</u> 4 but not multiples of 5?

(b) How many positive integers strictly less than 2026 are multiples of 3 <u>or</u> 4 but not multiples of 5?

5. Consider the following *logic table*, with entries T (True) and F (False).

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

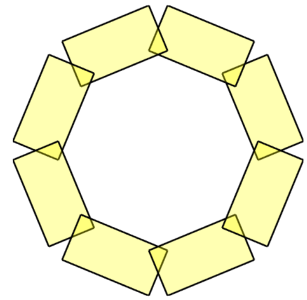
(a) Based on the table, what is the probability that the k-times nested statement

$$\underbrace{(((\cdots(}_{k\text{ times}} s_0 \to s_1) \to \cdots s_{k-1}) \to s_k)$$

is T? Here $k \geq 1$ and each of s_0, s_1, $\ldots$, s_k are either T or F.

(b) What is probability that the infinitely nested statement is true?

6. Salvador has a paint brush of width 4 centimeters. Each thick "segment" made with the brush is in fact a rectangle of width 4 centimeters and some number of centimeters in length. Salvador paints eight legs of an "octagon", each a 4 centimeter by 10 centimeter rectangle as shown in the figure. The short sides of the rectangles have midpoints which coincide. What is the area of the genuine octagon formed inside the painted rectangles (the inside area which is not painted)?



7. Suppose u and v are real numbers with $u \geq 1$, and consider the following expression.

$$f(u,v) = uv + \sqrt{(u^2-1)(v^2+1)} + \sqrt{\left(uv + \sqrt{(u^2-1)(v^2+1)}\right)^2 + 1}$$

(a) Write this expression as $f(u,v) = g(u)h(v)$, where $g(u)$ is a function solely of u and $h(v)$ is a function solely of v. That is, separate variables.

(b) Find a solution to the nonlinear system of equations

$$f(u,v) = 1, \qquad u^2 + v^2 + uv = \tfrac{7}{8}.$$

That is, find specific values of u and v which obey both equations simultaneously. Is your solution unique? *Hint*: first express the solutions to $f(u,v) = 1$ in terms of a single parameter $t = h(v)$.

8. MATLAB (software invented at UNM by Cleve Moler) represents a computer number as

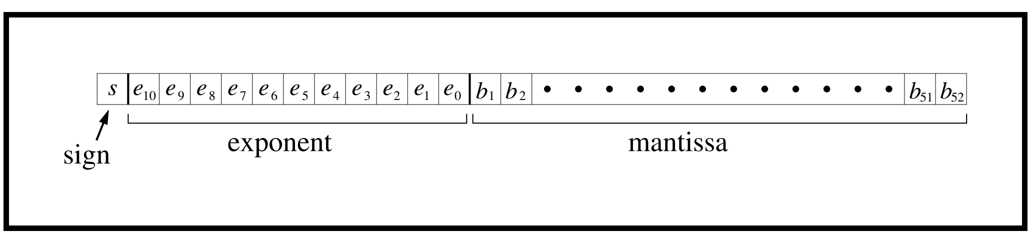
$$x = (-1)^s (1.b_1 b_2 b_3 \cdots b_{52})_2 \times 2^{F-1023}.$$

This representation corresponds to a 64-bit string in computer memory, as shown in the figure. Each "bit" is either a 0 or 1. The *sign* bit s determines whether the number is positive or negative. The *exponent* F is stored as an 11-bit string $e_{10}e_9e_8e_7e_6e_5e_4e_3e_2e_1e_0$, and corresponds to the number

$$F = (e_{10}e_9 \cdots e_0)_2 = e_{10}2^{10} + e_9 2^9 + \cdots + e_0 2^0.$$

The *true exponent* $F - 1023$ differs from F by an *exponential bias* of 1023. The *mantissa* is a 52-bit string $b_1 b_2 \cdots b_{52}$ which defines the number

$$(1.b_1 b_2 \cdots b_{52})_2 = 1 + \frac{b_1}{2} + \frac{b_2}{2^2} + \cdots + \frac{b_{52}}{2^{52}}.$$



(a) What is the allowed range for F (its smallest and largest possible values)?

(b) What is the allowed range for $(1.b_1 b_2 \cdots b_{52})_2$? Give compact answers.

(c) Suppose that x is stored in computer memory as the following string, with two zeros among 62 ones.

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0111 1111 1110 1111 1111 1111 1111 1111 1111 1111 1111 1111 1111 1111 1111 1111
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Give a compact expression for x.