UNM-PNM Statewide High School Mathematics Contest LVII
Round-2, 8 February 2025, 13:00-16:30

Name: ____________________ **Email:** ____________________ **Grade:** ___

School: ____________________ **Teacher:** ____________________

Instructions:

- *Please write your name on each of your work pages.*
- *Please start each problem on a new page (either front or back).*
- *Please number your pages.*
- *Please no phones or calculators.*

For the graders (please do not mark below the line)

#1		
#2		
#3		
#4		
#5		
#6		
#7		
#8		

TOTAL: ____________ ____________

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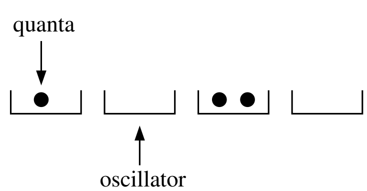
1. A two-pan balance scale is used to identify a counterfeit coin via weight comparisons.

(a) Nine coins are identical in appearance, but one is counterfeit. The counterfeit coin is heavier than the others. How can you guarantee identification of the counterfeit coin with at most two scale comparisons?

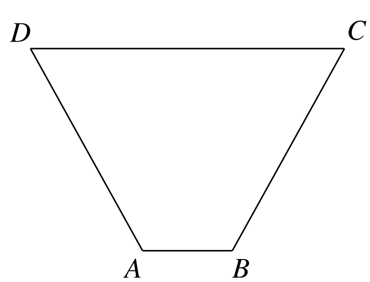
(b) Find the least number of scale comparisons necessary to guarantee identification of one heavy counterfeit coin among 27 coins which are identical in appearance.

2. Suppose M quanta of energy are distributed in N one-dimensional quantum mechanical oscillators. A key question (in quantum statistical mechanics!) is how many possible ways can the indistinguishable quanta be distributed. What is the answer for the scenario $M = 3$ and $N = 4$, with one possible state shown in the figure?



3. Simplify the expression $\sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}}$.

4. In the figure the isosceles trapezoid $ABCD$ has side AB parallel to side CD, sides AD and BC are of equal length, and the diagonals AC and BD are perpendicular. If the length of the side AB is 1 and the length of the side AD is 5, find the length of the side CD.



5. A sequence $a_1, a_2, a_3, \ldots$ of numbers is said to be an *arithmetic progression* if each term (other than the first) is the previous term plus a fixed number r. This means $a_2 = a_1 + r$, $a_3 = a_2 + r$, and, generally, $a_n = a_{n-1} + r$ for $n > 1$. For example, the sequence $3, 7, 11, 15, 19, 23, 27, 31, 35, \ldots$ is an arithmetic progression with $r = 4$ (presuming the pattern continues). The first three terms of an arithmetic progression of positive numbers are

$$a_1 = \tan x, \qquad a_2 = \cos x, \qquad a_3 = \sec x,$$

for some angle x in the first quadrant.

(a) What is the angle x?

(b) What is r?

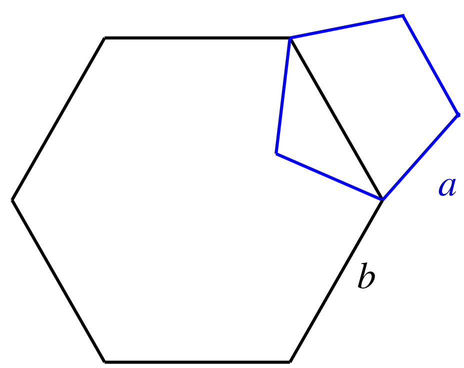
(c) What position does $\cot x$ occupy in the sequence?

6. In how many ways can one choose a black square and a white square on an 8×8 chessboard, so that the chosen squares do not lie on the same row or on the same column?

7. In the figure both the hexagon and pentagon are regular; each has sides which are equal.

(a) Show that $\cos(\frac{1}{5}\pi) = \frac{1}{4}(\sqrt{5} + 1)$ and $\cos(\frac{2}{5}\pi) = \frac{1}{4}(\sqrt{5} - 1)$.

(b) Using the results from **(a)**, find the ratio of the area of the large hexagon (with side length b) and the area of the small pentagon (with side length a).



8. **(a)** If the expression

$$(((x-2)^2 - 2)^2 - 2)^2,$$

with three pairs of parentheses, is multiplied out, what is the coefficient of x^2?

(b) If the expression

$$(\cdots(((x-2)^2 - 2)^2 - 2)^2 - \cdots - 2)^2,$$

with 2025 pairs of parentheses, is multiplied out, what is the coefficient of x^2?