

UNM-PNM Statewide High School Mathematics Contest 2024-2025
In-person Round 1, 22-25 November 2024

Participant information and responses should be entered online at

<https://esurvey.unm.edu/opinio/s?s=156104>

Instructions (please read carefully)

- Over a **continuous three-hour period**, please attempt as many of the following **ten** problems as possible.
- All **entered** answers should be integers. Please read each problem carefully. Some have specific instructions for reporting entered answers.
- For entered answers, write 2, not 2.0 or two. Do not include commas; write 100421, not 100,421.
- **No calculators or external sources permitted.**

Thank you for participating!

Student (LAST name, FIRST name): _____

Student or teacher email address: _____

Grade level: _____

School or club: _____

Teacher or coach: _____

1. Which is greater $\frac{2023}{2024}$ or $\frac{2024}{2025}$? As your answer report the denominator of the larger number.

ENTERED ANSWER:

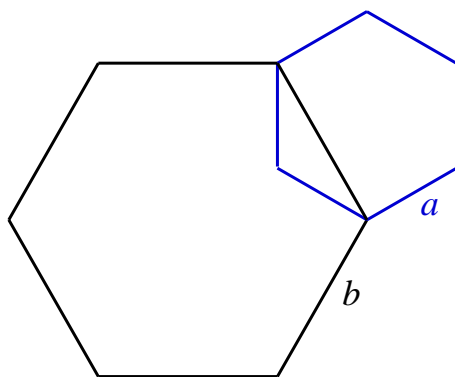
2. There are 24 four digit numbers such that in each the digits 2, 4, 5, and 7 appear exactly once (for example, 5472 or 4275 are such numbers). Among these numbers, there is one which is a multiple of another one. Find both numbers, and report their sum as your answer.

ENTERED ANSWER:

3. Cora and Ernie play the following game. Starting with three piles, each with 7 stones, they take turns picking stones from the three piles. Per turn a player can take any number of stones from one pile, but must take at least one stone and must take stones only from the same pile. The winner is the last person to take a stone. Cora is the first player to pick stones. One player can always win, which one? If Cora, then report 1 as your answer; if Ernie, then report 2 as your answer.

ENTERED ANSWER:

4. In the figure both hexagons are regular, that is each hexagon has sides which are equal. Find the ratio of the area of the large hexagon (with side length b) and the area of the small hexagon (with side length a).



ENTERED ANSWER:

5. Let $x = \sqrt{1 + \sqrt{1 + \sqrt{1}}}$. Evaluate the expression $x^4 - 2\sqrt{2}$.

ENTERED ANSWER:

6. Each of the four friends, Mario, Andrea, Jess, and Paul, have dyed their hair a unique color: either blue (color number 1), green (color number 2), pink (color number 3), or purple (color number 4). They stand in a circle. The friend with the blue hair, who is neither Mario nor Andrea, is standing between the friend with the purple hair and Jess. The friend with the pink hair is standing between the friend with the green hair and Mario. What is each friend's hair color? Fill in the table below, and report as your answer the four-digit number from the last row, for example 3241 would mean Mario has pink hair, Andrea has green hair, Jess has purple hair, and Paul has blue hair.

name	Mario	Andrea	Jess	Paul
color				
color number				

ENTERED ANSWER:

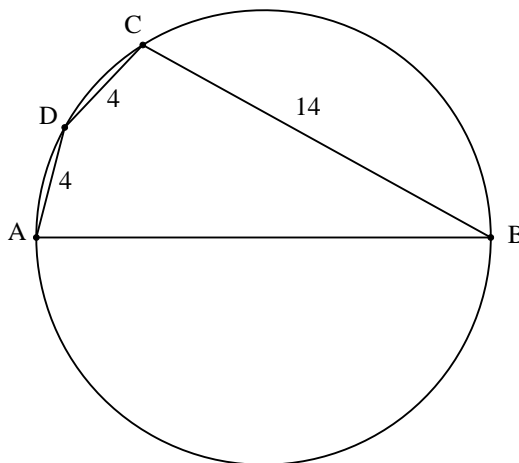
7. In this problem *life span* means *average life span* in years. The life span of a bowhead whale is 8 times that of a brown bear, which is 11 years shorter than that of a western gorilla, which is three times the life span of a wolverine, which is 8 years longer than that of a wild turkey, which is 10 years shorter than the life span of a tiger, which is a quarter of the life span of an elephant, which is equal to one sixth of the quantity formed as -11 plus the sum of all seven life spans. What is the life span of a wild turkey?

ENTERED ANSWER:

8. A perfect square is a whole number that is the square of another whole number; for example, $100 = 10^2$ and $169 = 13^2$ are perfect squares, but 56 and 2024 are not. Find the sum of all non-negative integers n such that $n^2 - 11n + 25$ is a perfect square.

ENTERED ANSWER:

9. In the figure the side AB of the quadrangle $ABCD$ is a diameter of the circumcircle, with the length of BC equal to 14 and the lengths of CD and DA both equal to 4. What is the radius of the circle?



ENTERED ANSWER:

10. For a point in the (x, y) -plane consider the transformation

$$u = \frac{1}{2}x(1 + r^{-2}), \quad v = \frac{1}{2}y(1 - r^{-2}),$$

where $r = \sqrt{x^2 + y^2}$. The transformation maps circles $x^2 + y^2 = r^2$ in the (x, y) -plane into ellipses $(u^2/a^2) + (v^2/b^2) = 1$ in the (u, v) -plane. For the ellipse in the (u, v) -plane with major diameter $2a = \frac{10}{3}$ and minor diameter $2b = \frac{8}{3}$, find the radii of all circles which are mapped to this ellipse. For your answer, report 3 times the sum of all these radii.

ENTERED ANSWER: