

UNM - PNM STATEWIDE MATHEMATICS CONTEST L

November 3-6, 2017 First Round Three Hours

1. Let A, B be two sets of integers. $A = \{2, 0, 1, 7\}$ and $B = \{x \mid -x \in A, 2 - x^2 \notin A\}$. Find the sum of all the integers in B .
2. Right now my age is the sum of the ages of my two sisters. Two years ago I was twice as old as my middle sister. Eight years from now my younger sister will be the age I am now. How old am I?
3. Four consecutive sides of an equiangular hexagon have lengths 2, 9, 5, and 7. Find the lengths of the two remaining sides.
4. Find all the prime numbers p such that $4p^2 + 1$ and $6p^2 + 1$ are prime numbers.
5. Kayla loves almond butter but cannot afford to buy her own. Her roommate Diane bought a 200 mL jar. Kayla decides to steal some almond butter from Diane's jar every day immediately after Diane eats, but to make sure that Diane doesn't know, she will steal at most 20 mL but no more than one quarter of what is left in the jar. If Diane eats 30 mL each day or finishes the jar, how much almond butter does Kayla steal?
6. Consider the table:

1	2	3							
4	5	6	7	8	9				
10	11	12	13	14	15	16	17	18	

If we continue to add rows so that the next row has 3 more consecutive terms than our current row, in which row is 2017?

7. Two hundred people have come to hear a lecture on the benefits of chocolate. A survey is given out at the beginning as to the chocolate preferences of the audience members. Suppose 120 people like dark chocolate, 100 people like milk chocolate, 60 people like white chocolate, 40 people do not like any kind of chocolate and 10 people like all three kinds of chocolate, how many people like at least 2 types of chocolate?
8. Every week Iris chooses a fruit of the week from the following fruits: apple, banana, orange and mango. She decides that each week, she will pick a fruit which is different than previous week's fruit and she will choose one randomly from the remaining three. Now suppose in the first week of the school year, Iris chose apple. What's the probability that she will pick apple again in the seventh week of the school year? Provide your answer in the simplest form.
9. On square $ABCD$, points E and G are placed on \overline{AD} so that G is between E and D and F and H are placed on \overline{BC} so that F is between B and H and the lengths $BE = EF = FG = GH = HD = 52$ units. Find the area of $ABCD$.
10. Let a, b be real numbers and $f(x) = ax + b$. Suppose for any $x \in [0, 1]$, we have $|f(x)| \leq 1$. Find the maximum value of ab .