

UNM - PNM STATEWIDE MATHEMATICS CONTEST XLVII

November 7-10, 2014 First Round Three Hours

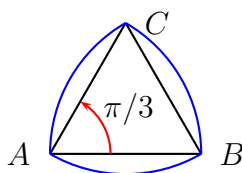
1. In the solution of the following 6 by 6 sudoku puzzle, what are the numbers a, b, c and d . By a solution of the puzzle is meant a placement of the numbers 1 to 6 in the empty squares so that each row, each column and each 2×3 box contains the same number only once.

		2	5		
	3			1	
2					4
1	a	b	c	d	2
	2			3	
		5	4		

2. Simplify the expression

$$Q = \left(\frac{1}{(a+b)^2} \left(\frac{1}{a^2} + \frac{1}{b^2} \right) + \frac{2}{(a+b)^3} \left(\frac{1}{a} + \frac{1}{b} \right) \right) a^2 b^2$$

3. Find the smallest positive integer with precisely 42 factors.
4. Find the exact numerical value of $a = \sqrt{|12\sqrt{10} - 49|} - \sqrt{|12\sqrt{10} + 49|}$
5. Find the area of the figure which is the intersection of three discs of radius 1 with centers at the three vertices of an equilateral triangle with sides of length 1.



6. The decimal $.3\overline{7027}$ can be expressed in the form $\frac{m}{n}$ where m and n are relatively prime. Find $n - m$.
7. Find the exact value of $\sin \alpha$ if $\tan(\alpha/2) = \sqrt{3}/2$.
8. In a triangle with sides of lengths 5, 6 and 9 is drawn a circle touching the two shorter sides with a center on the longest side. Find the radius of the circle.
9. An urn contains black and white balls. It is known that we can draw at random with equal probability two balls of the same color or two balls of different color. What is the number of balls that must be in the urn so that there are at least 2014 and at most 2114 balls.
10. Consider the two arithmetic sequences
 (a) 17, 21, 25, 29, ... and (b) 16, 21, 26, ...
 Find the sum of the smallest 40 numbers that appear in both sequences.