UNM - PNM STATEWIDE MATHEMATICS CONTEST XLV

February 4, 2013 Second Round Three Hours

1. A fox running at its top speed can run 200m in 10sec, while a squirrel running at its top speed can run 300m in 50sec. The fox sees the squirrel 21 meters ahead and starts chasing it. Suppose that both the fox and the squirrel are running along a straight line in the same direction maintaining their respective top speeds throughout the chase. When will the fox catch the squirrel?

2. Solve the equation $(6x-1)^2(x+1)-(6x+1)^2(x-1)=14$.

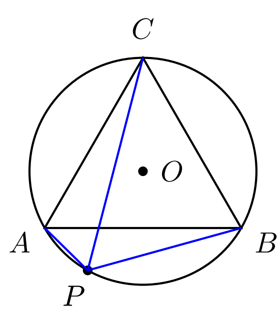
3. For which positive integers n is the triple $\left(\frac{1}{n}, \frac{1}{n+1}, \frac{1}{n+2}\right)$ the sides of a triangle.

4. In a volleyball tournament every team plays one game against each of the remaining teams. Show that regardless of the set schedule, at every moment of the tournament there are at least two teams who have played the same number of games.

5. To every pair of positive real numbers x and y, we assign a positive real number $x*y$ satisfying the two properties that $x*x=1$ and $x*(y*z)=(x*y)\cdot z$ where $\cdot$ represents the standard multiplication of real number. Determine $61*2013$.

6. Show that there is no solution to THREE + FIVE = EIGHT if different letters denote different digits.

7. Show that the sum of the squares of the distances from the vertices to a fixed point on the circumscribed circle of an equilateral triangle is independent of the point on the circle.



8. Find all non-negative integer numbers n and m such that $3^n=1+2^m$.

9. Find all points $P(x,y)$ in the unit square $R=[0,1]\times[0,1]$ such that the difference between the two coordinates is at most 1/2, i.e., $|x-y|\le 1/2$.

10. Two numbers are picked at random from the interval $(0,1)$. What is the probability that the difference between the two numbers is at most 1/2?