UNM - PNM STATEWIDE MATHEMATICS CONTEST XLV

November 2-5, 2012 First Round Three Hours

1. Find the closest integer to $a + b$ if

$$a = \frac{1}{4} + \frac{3}{8} + \frac{5}{12} + \cdots \frac{1005}{2012}$$

and

$$b = \frac{5}{8} + \frac{7}{12} + \frac{9}{16} + \cdots \frac{1009}{2016}.$$

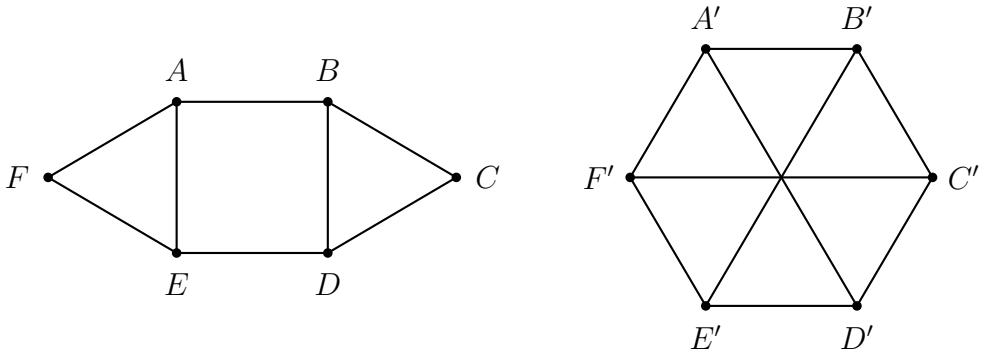
2. What is the smallest number of seats in a large auditorium that must be occupied in order to be certain that at least two people share the same first and last initials?

3. Using a line through one of its vertices a triangle is cut in two isoceles triangles. If the measure of the angle opposite one of the congruent sides in the first triangle is 40 degrees, what are the possible measurements (notice the plural!) for the angles opposite the congruent sides on the adjacent isoceles triangle?

4. Suppose that only eight tiles are left in the scrabble bag and the letters on the tiles spell CALCULUS. How many ways can you choose two tiles?

5. Suppose $0 \leq a_i < n$ for $i = 0, 1, 2, \ldots, r$. The number $(a_r a_{r-1} \cdots a_1 a_0)_n$ represents the number $a_r n^r + \cdots a_1 n + a_0$ in base n. For example, $(102)_{13}$ is the base 13 representation of $1 \cdot 13^2 + 0 \cdot 13^1 + 2 \cdot 13^0 = 13^2 + 2 = 171$. In which bases n is $(11)_n$ a perfect square?

6. Using unit squares and equilateral triangles whose sides are of length one, you can form a convex hexagon with a unit square and two equilateral triangles whose sides are of length one or 6 equilateral triangles whose sides are length one as shown below.



How many unit squares and unit equilateral triangles can be used to construct a convex hexagon whose sides are all of length n?

7. Let ΔABC be an equilateral triangle whose side is of length 1 inch. Let P be a point inside the triangle ΔABC. Find the sum of the distances of P to the sides of the triangle ΔABC.

8. Nathan just aced his math test and he is hoping that his parents will reward him for his performance. Nathan's parents decide that Nathan deserves a reward for his hard work; however, they like to add a little bit of chance to the reward. Nathan's parents have 5 crisp new 5 dollar bills and 5 crisp new 10 dollar bills. They tell Nathan that he has to divide the bills into two groups. Nathan's parents explain that after blindfolding Nathan they will place each group into a brown bag, after shuffling the bills. Then they will place one bag on the right hand side of a table and one on the left hand side of the table. He will choose one of the bags without examining them and then he will reach in and grab one of the bills. What is the highest probability that Nathan can achieve in picking a 10 dollar bill out of all possible groupings of the bills?
9. A boat is traveling against the flow of a river. Suppose the river is flowing at a constant speed and the boat maintains a constant speed with respect to the river while traveling in either direction along the river. At a certain moment of time a blow-up ball falls off the boat and starts floating down the river. 20 minutes after the ball fell into the water this was noticed and the boat reversed its direction and started going down the river chasing the ball. How long was the ball in the water before it was retrieved?
10. Let ΔABC be an equilateral triangle. Find all points in the plane such that the distance from any such point to one of the vertices equals the sum of the distances to the remaining two.