UNM - PNM STATEWIDE MATHEMATICS CONTEST XLIII

February 5, 2011 Second Round Three Hours

The order of the problems on each of the pages is not indicative of their difficulties.

(1) Find the smallest positive integer n such that every digit of $45n$ is 0 or 4.

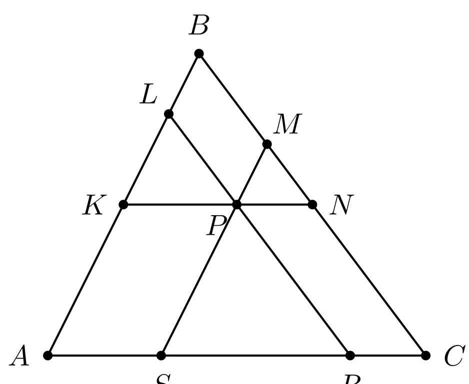
(2) At an artisan bakery, French tortes are 52 dollars, almond tarts are 12 dollars and cookies are one dollar each. If Alex has 400 dollars to purchase exactly 100 of these items for a party and he buys at least one of each item, how many of each bakery item does he purchase? Only whole pieces of the bakery items can be purchased.

(3) In the following "equation" each letter represents a digit (between 0 and 9). Different letters represent different digits and S is not 0. Determine the digit represented by each of the used letters so that the addition is correct.

$$\begin{array}{rr} & STORE \\ + & STORE \\ & STORE \\ \hline & TEASE \end{array}$$

medskip

(4) P is a point inside the triangle $\triangle ABC$. Lines are drawn through P parallel to the sides of the triangle. The areas of the three resulting triangles $\triangle PMN$, $\triangle PLK$ and $\triangle PRS$ are 9, 25 and 81, respectively. What is the area of $\triangle ABC$?



(5) (a) Let n and d be integers. Show that there are infinitely many integers m such that $n^2 + md$ is an exact square.

(b) An arithmetic progression is a sequence of numbers of the type

$$a,\ a+d,\ a+2d,\ a+3d,\ a+4d, \ldots,$$

where d is the common difference between two successive terms. Prove that if an arithmetic progression of integers contains an exact square (of an integer number), then it contains an infinite number of exact squares.

(6) Find the smallest possible ratio of the radii of two concentric circles centered at a point inside a given triangle so that one of the circles contains while the other is contained in the given triangle. Thus, each of the vertices of the triangle lies inside or on the larger circle, while no point of the smaller circle is outside the triangle.

(7) Let z_1, z_2 and z_3 be the roots of the polynomial $Q(x) = x^3 - 9x^2 + 1$. In other words, $Q(z_1) = Q(z_2) = Q(z_3) = 0$. If $P(x) = x^5 - x^2 - x$, what is the value of $P(z_1) + P(z_2) + P(z_3)$?

(8) A very wild ant is moving in a plane with a fixed Cartesian (orthogonal) coordinate system.

a) Suppose the ant is moving parallel either to the x- axis or the y-axis. Suppose the ant travels 3 centimeters per second parallel to the y axis direction (in either direction) and 2 centimeters per second when moving parallel to the x axis (in either direction). If the ant starts at the origin, what region of the plane can the ant reach in one second?

b) Suppose the ant can travel 3 centimeters per second when traveling *on the y-axis* and 2 centimeters per second in any other direction when off the y-axis. If the ant starts at the origin, what region of the plane can the ant reach in one second?

(9) The faces of a solid figure are all triangles. The figure has 11 vertices. At each of six vertices, four faces meet and at each of the other five vertices, six faces meet. How many faces does the figure have?

(10) Nine scientists are working on a secret project. They wish to lock up the documents in a cabinet so that the cabinet can be opened when and only when *five or more of the scientists are present*. For this purpose a certain number of locks are installed on the cabinet and each of the scientists is given keys to some of these locks. Each key can open exactly one lock. Thus, for the cabinet to be opened (i) any five of the scientists have to be present and (ii) the keys to all of the locks on the cabinet have to be among the set of all keys given to the present five scientists. What is the smallest number of locks needed?