UNM - PNM STATEWIDE MATHEMATICS CONTEST

October 31 - November 2, 2020 First Round Three Hours

1. Twenty lemons cost the same number of coins as the number of lemons that you could buy for 500 coins. How much do 10 lemons cost?

2. The lengths (in centimeters) of of each of four triangles I, II, III and IV are as follows:

$$I : 20, 21, 29 \qquad II : 3, 4, 5$$

$$III : 4, 7\frac{1}{2}, 8\frac{1}{2} \qquad IV : 7, 8\frac{1}{2}, 11\frac{1}{2}$$

Which of these are right-triangles (that is, triangles having a right angle)?

3. A jar contains 2020 balls. Each ball may have one or more colors on it (for example, a ball could be completely colored red, another could be colored red and blue, and another could be colored green, yellow and orange). The only information we have is that the number of balls with red on them (for example, a ball which is colored in both blue and red) is a multiple of 7. Similarly, the number of balls with blue on them is a multiple of 9, and the number of balls with yellow on them is a multiple of 15. What is the maximum number of balls colored both red and blue but not yellow?

4. 90 students attended the Public Lecture of the statewide UNM-PNM math contest last year. Some of these students arrived early and had snacks which were served before the lecture started. Donuts were eaten by 32 students, cupcakes were eaten by 61 students, and sandwiches were eaten by 29 students. 28 students had both a donut and a cupcake, 16 students had both a cupcake and a sandwich, and 10 students had both a donut and a sandwich. Only 6 students ate all three snacks. How many students did not eat any snack?

(Note: The online version had a typo and stated that sandwiches were eaten by 19 students. We gave everyone full credit for this problem.)

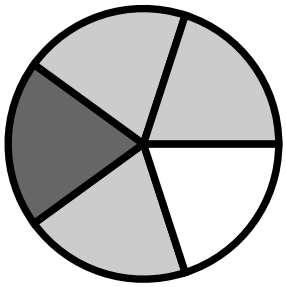
5. Suppose $x_1 = a$ and $x_{n+1} = x_n + b$. For what values of a, b do we have $x_0 + \cdots x_n = n^2$?

6. What are the last two digits of 6^{2020}?

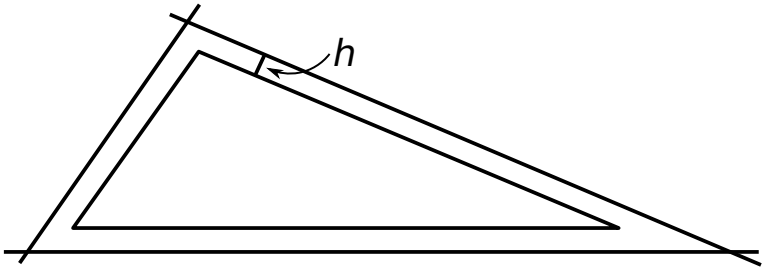
7. Four excellent mathematics school students are walking down the street in downtown Gallup, NM, when they see an absolutely flagrant violation of the New Mexico Rules of the Road. As the car is speeding away, the first student notes the license plate starts with "TX" and is followed by four digits; the second student notes the next two digits are identical to each other; the third student notes the *last* two digits are identical to each other; and the final student notes the four digits form a perfect square. What is the license plate number?

8. A disc is divided into p sectors, where p is a prime. Each sector is painted using one of n colors. How many ways are there to paint the disc if $n = 5$ and $p = 7$? Count all configurations which are rotations of each other as one case.

   For reference, below is an example of a disc with 5 sectors and 3 colorings (dark gray, light gray, white).



9. A triangle has area S and perimeter P. Each of the lines forming the sides of the triangle are moved outward a distance h in the direction that's perpendicular to the corresponding side of the triangle, as in the figure below. What is the area S' of the new triangle when $S = 100$, $P = 50$, and $h = 1.6$?



10. Given a positive integer n, we know that the number of non-negative integer solutions to

$$x_1 + x_2 + \cdots x_k = n$$

is $\binom{n+k-1}{k-1}$, where $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ and $n! = n \times (n-1) \times (n-2) \cdots 2 \times 1$. For example consider finding non-negative integer solutions to,

$$x + y = 3.$$

Here $n = 3, k = 2$, and so we have $\binom{4}{1} = 4$ solutions (these are $x = 0, y = 3$, $x = 1, y = 2$, $x = 2, y = 1$, and $x = 3, y = 0$).

Now we want to find the number of non-negative integer solutions to

$$a + b + c + d = 16$$

such that $2 \leq a \leq 5, \quad 1 \leq b \leq 8, \quad 0 \leq c \leq 6, \quad 3 \leq d \leq 8$. How many such solutions are there?