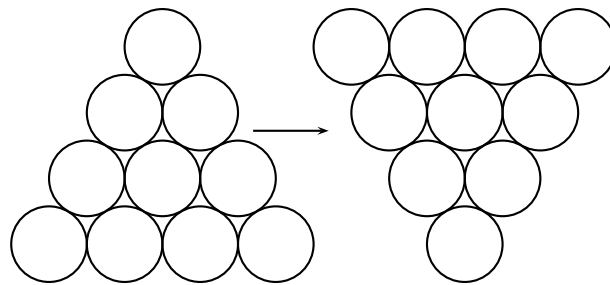


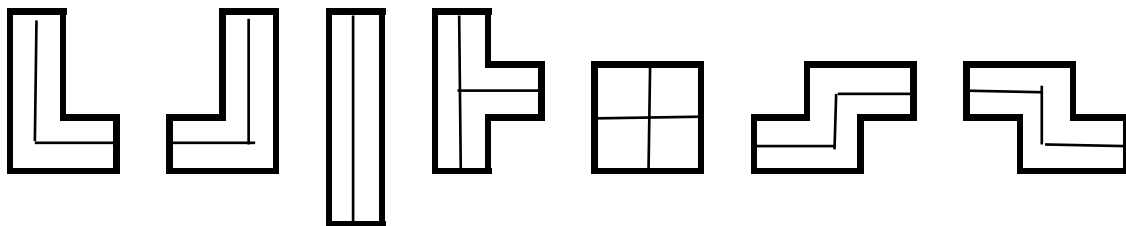
UNM - PNM STATEWIDE MATHEMATICS CONTEST XLVIII

November 6-9, 2015 First Round Three Hours

1. Among 50 students in the world languages club, 20 know Spanish, 15 know French and 10 know German. If 5 know both Spanish and French and 2 know French and German but none know both Spanish and German, how many students don't know any of these languages?
2. A team of 5 students are chosen to work on a research project lead by a chemical engineer. If 4 chemistry students, 3 physics students and 2 math students express interest in working with the professor, how many ways can the professor choose 5 of the students if he wants at least 2 chemistry students on the team?
3. A student is offered two different after school jobs. One pays \$8 an hour, the other pays \$1 for the first hour, but the hourly rate increases by \$1 dollar for each additional hour worked. At least how many hours must the student work, so that it will make it worth his while to take the second job?
4. Let $f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$ and $g(x) = x^3 + x^2 - 1$, find polynomials $q(x)$ and $r(x)$ with integer coefficients such that $f(x) = g(x)q(x) + r(x)$ and the degree of $r(x)$ is smaller than 3.
5. Ten congruent discs are packed forming an "equilateral triangle" in the xy coordinate plane whose base is parallel to the x -axis and the third vertex is above the base. You are asked to slide some of the discs to form a new "triangle" from the old one such that the new "triangle" has its base again parallel to the x -axis but now the remaining vertex is below the new base. A picture is provided below to assist you with the orientation of the initial and final "triangles". What is the minimal number of slides a student must perform to produce this new "triangle"?



6. In the game of tetris, there are 7 distinct polyomino pieces each constructed from four unit squares.



Each piece can be rotated 90, 180, or 270 degrees. List all values of n for which we can construct a rectangle of area $4n$ using *distinct* tetris pieces so that no pieces overlap and the rectangle has no holes?

7. The bottom of a swimming pool needs to be retiled. The surface that needs new tiles is a rectangle with sides of lengths 40.04m by 25.20m. In order to finish the job faster the contractor wants to use as large tiles as possible. They can order any size square tile of integer length in centimeters. What is the size of the tile the contractor ordered. Note: One meter has 100 centimeters, 1m=100cm.
8. A triangle has sides all of whose lengths are integers. If two of its sides are 2 and 4, what are the possible lengths of its other side?
9. What must the radius of a sphere be so that its volume is equal to the volume of a triangular prism of height two and whose base is an equilateral triangle whose sides are all 1?
10. Four ants A , B , C and D are out in a flat field when they realize that they cannot remember where their den is located. Each one of them decides to rely on the memory of the ant it sees directly ahead, so accordingly it decides to always walk directly towards this ant. It just so happens that at a certain moment the ants are at the corners of a square with sides of length 2 meters so that $AB = BC = CD = DA = 2\text{m}$ (AB denotes the distance between ants A and B etc.), with ant A looking directly at ant B , who is looking at ant C , who is looking at ant D , who is looking at ant A . Suppose each of the ants moves at a constant speed of 3 cm/sec with ant A always moving directly towards ant B , who is moving towards C , who is moving towards ant D , who is moving towards ant A . How long will it take before the four ants meet if they meet at all?