Spring 2022 UNM-PNM Math Contest

1. Two people each have jobs. The first person works 40 hrs/week at x dollars per hour. The second person works 30 hrs/week at y dollars per hour. If they switch so that the first person works 30 hrs/week and the second person works 40 hrs/week, they will increase their combined income by \$100 per week. How much per hour does the second person make?

2. Suppose a car going 65 mph is traveling North parallel to a train going 50mph. Once the front of the car is as far north as the back of the train, it takes another three minutes for the front of the car to be further North than the front of the train. How long is the train?

3. In how many ways can the number 2022 be written as the sum of consecutive integers?

4. Suppose $p > 0$. Find the smallest positive ϵ such that

$$\frac{p}{1+p} + \frac{p}{(1+p)(1+2p)} + \cdots + \frac{p}{(1+314158p)(1+314159p)} + \epsilon$$

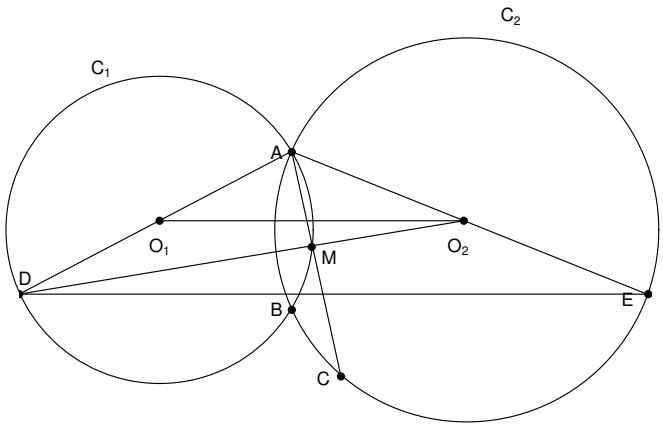
   is a whole number.

5. Find all the solutions to the equation

$$\sqrt[3]{25x(2x^2+9)} = 4x + \frac{3}{x}$$

6. Consider words consisting of letters from the alphabet $\{a, c, g, t\}$. How many words of length 8 are there where the first and letters are both a, and no two consecutive letters are the same?

7. Circle C_1 has center O_1 and radius 1. Circle C_2 has center O_2 and radius $\sqrt{2}$. The circles intersect at points A and B (see diagram). Let AC be the chord of C_2 that is bisected by C_1. Find the length of AC given that O_1 and O_2 are 2 units apart.



8. How many whole numbers between 1 and 2022 (inclusive) are perfect squares?

9. Suppose A, B, C are points in a plane and $\angle ACB = \theta$. Describe the set of all the points X satisfying $|AX|^2 + |AB|^2 = |AC|^2$

10. In a long line of people waiting to buy the latest edition of the popular magazine *Shiprock Mathematics Enthusiast*, each person either has a \$10 bill or a \$5 bill. If there are n people with \$5 bills and m people with \$10 bills, what is the probability the cashier will never run out of change if the cost of the magazine is \$5?