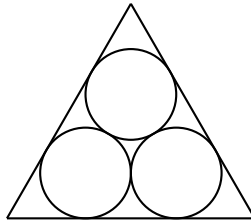


UNM - PNM STATEWIDE MATHEMATICS CONTEST XLVI

November 1-4, 2013      First Round      Three Hours

1. What are the last two digits of  $7^{2013}$ ?
2. A quadratic equation  $ax^2 + ax + b = 0$  has two solutions  $x_1$  and  $x_2$ . What is the sum of the squares of the two solutions?
3. Simplify  $\frac{2^4 + 2^4 + 2^4 + 2^4}{4^3 + 4^3 + 4^3}$ .
4. Suppose that on Halloween night three ghosts which are either spooks or poltergeists are having a conversation. Simple spooks always tell the truth but poltergeists always lie.  
 Ghost 1: "Ghost 2 is a poltergeist."  
 Ghost 2: "Ghost 1 and 3 are either both spooks or both poltergeists."  
 What is Ghost 3?
5. A palindrome is a number that reads the same forwards and backwards. How many 11 digit palindromes are there that include at most 2 of the same digit?
6. Three trees in a perfectly flat forest are growing on the same line. The heights of the first and third trees are  $a$  and  $b$  respectively. What is the height of the middle tree if its top lies both on the line connecting the top of the first tree and the bottom of the third tree and the line connecting the top of the third tree with the bottom of the first tree?
7. Find the largest sum of all integer numbers  $x$  and  $y$  such that
 
$$x^2y - xy^2 - xy - 2x + 2y + 1 = 0.$$
8. Suppose three circles of radius one are packed inside an equilateral triangle so that every two circles are tangent to each other and the sides of the triangle are tangent to the circles; see figure below. What is the area of the triangle?



9. Simplify as much as possible the expression
 
$$\left(\frac{x(x^3 + 2y^3)}{x^3 - y^3}\right)^3 + \left(\frac{y(y^3 + 2x^3)}{y^3 - x^3}\right)^3.$$
10. The currency in Pairica uses bills of denominations 2, 4, 6,..., 100. In other words, there is a bill for every even integer number between 1 and 100. An urn contains 31 arbitrary bills of different denominations. You are to name a number and if the urn contains two bills whose sum is the number you named you can take these two bills. What is the largest amount you can take with absolute certainty?