UNM - PNM STATEWIDE MATHEMATICS CONTEST XLIV

November 4-7, 2011 First Round Three Hours

1. How many positive integer numbers less than or equal to 2011 are multiples of both 3 and 5 but not multiples of 8?

2. Three boxes are presented to you. One contains $1000, the other two are empty. Each box has a clue written on it as to its contents and only one message is telling the truth, the other two are lying. If the first box says, "The money is not here", the second box says "The money is in the first box" and the third box says, "The money is not here", which box has the money?

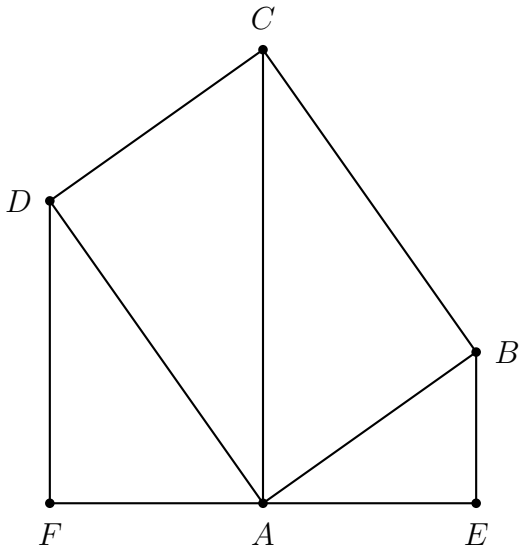
3. It takes a horse and a goat two hours to eat 20 pounds of hay. If it takes the horse three more hours than the goat to eat 20 pounds of hay, how long does it take the horse to eat the 20 pounds of hay?

4. The door to the computer room at a school has a keycode. The combination is a sequence of 5 numbers. A student forgot his code. However, he did remember five clues. These are what those clues were:
   (a) The fifth number plus the third number equals fourteen.
   (b) The fourth number is one more than the second number.
   (c) The first number is one less than twice the second number.
   (d) The second number plus the third number equals ten.
   (e) The sum of all five numbers is 30.
   What were the five numbers and in what order?

5. Thirty bored students take turns walking down a hall that contains a row of *closed* lockers, numbered 1 to 30. The first student opens all the lockers; the second student closes all the lockers numbered 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30; the third student operates on the lockers numbered 3, 6, 9, 12, 15, 18, 21, 24, 27, 30: if a locker was closed, he opens it, and if a locker was open, he closes it; and so on. For the i^{th} student, he works on the lockers numbered by multiples of i: if a locker was closed, he opens it, and if a locker was open, he closes it. What is the number of lockers that remain open after all the students finish their walks?

6. A frog makes 2 jumps, each 1 meter in length. The directions of the jumps are chosen independently and at random with equal chances for every direction to be chosen. What is the probability that the frogs final position is at most 1 meter from its starting position?

7. In the figure below, $ABCD$ is a rectangle. The points A, F, and E lie on a straight line. The segments DF, BE, and CA are all perpendicular to FE. The length of DF is 15 and the length of BE is 6. Find the length of FE.



8. For each real number x , let $g(x)$ be the minimum value of the numbers $6x + 3$, $2x + 7$, $15 - x$. (For example if $x = 2$ then the three numbers are 15,11,13, so $g(2) = 11$.) Find the maximum value of $g(x)$.

9. The triangle $\triangle ABC$ has $AB = 7$ and the given ratio $BC/CA = 24/25$ of the lengths of the other two sides. What is the largest possible area for the $\triangle ABC$?

10. Let

$$x = .01234567891011 \cdots 998999$$

where the digits are obtained by listing the numbers 0-999 in order. What is the 2011^{th} digit to the right of the decimal place?