UNM–PNM STATEWIDE MATHEMATICS CONTEST XL

NOVEMBER 10, 2007 FIRST ROUND THREE HOURS

1. The obligatory questions about the current year:

   a. What is the last digit of 3^{2007}?

   b. What are the last *two* digits of 3^{2007}?

2. The real number x satisfying $x^3 = 2$ is called the cube root of 2 and is written $\sqrt[3]{2}$.

   a. Find the whole number part and the first *two* decimal places of $\sqrt[3]{2}$.

   b. What is $\sqrt[3]{2}$ rounded off to the nearest hundredth?

3. Suppose you have some dice that are six sided with the sides labeled one through six, and each side has an equal probability of occurring.

   a. If you roll two dice what is the probability that the sum of the two dice is six?

   b. If you roll three dice what is the probability that the sum of the three dice is eleven?

4. What are the slopes of the two tangent lines from the origin in the plane to the circle of radius 1/2 centered at (1,1)?

5. Maria has a jar of pennies. When she arranges the pennies in rows of 15 she finds that there are seven left over. When she arranges them in rows of 19 she finds that there are eight left over. What is the *smallest* number of pennies that could be in the jar?

6. Find the area of the triangle whose vertices, in the x-y plane, are $(2, 1), (8, 3), (6, 5)$.

7. The wheels on Heather's car are 23 inches in diameter. If she is traveling 60 miles per hour, how many times are the wheels rotating *per second* (rounded off to the nearest whole number)?

8. What is the polynomial with leading coefficient 1 and integer coefficients which has $\sqrt{2 + \sqrt{2}}$ as a root?