UNM–PNM STATEWIDE MATHEMATICS CONTEST XXXIX

NOVEMBER 11, 2006   FIRST ROUND   THREE HOURS

1. The obligatory question about the current year:

    a. Find the prime factorization of 2006.

    b. Find *all* whole number solutions to $x^2 - y^2 = 4012 = 2 \cdot 2006$.

2. An anagram of a word is another word (*not* necessarily belonging to the English language) made up of the same letters. So the word "nun" has three anagrams, namely "nnu," "nun," and "unn."

    a. How many anagrams are there of "math"?

    b. How many anagrams are there of "Mississippi"?

3. Suppose C is a circle of radius one.

    a. What is the largest number of points which can be placed on *or* inside of C so that no two are closer than one unit from one another?

    b. Draw a picture exhibiting the points for part (a).

4. Compute the first 3 decimal places of $\sqrt{7}$ (thus, for example, $\sqrt{11} = 3.31662\ldots$ so the correct answer would be 3.316).

5. Find the equation of the parabola which passes through the three points $(0, 1), (1, 4), (2, 9)$.

6. Two cars start together around a two mile race track. The first car is traveling 100 miles per hour while the second car is traveling 60 miles per hour.

    a. When do the cars cross the start line simultaneously?

    b. How long after the two cars start does the first car pass the second car?

7. You are given a square and are asked to divide it up into smaller squares. For example, a checker board consists of a big square divided into 64 smaller ones.

    a. Is it possible to cut the square into 6 smaller squares (not necessarily the same size)? If so, draw a picture.

    b. Is it possible to cut the square into 7 smaller squares (not necessarily the same size)? If so, draw a picture.

    c. Is it possible to cut the square into 8 smaller squares (not necessarily the same size)? If so, draw a picture.

8. One last problem to celebrate the year:

    a. What are the last three digits of 2006^5?

    b. What are the last three digits of 5^{2006}?