UNM–PNM STATEWIDE MATHEMATICS CONTEST XXXVI

NOVEMBER 8, 2003 FIRST ROUND THREE HOURS

1. A fair coin is tossed 10 times. What is the probability that exactly five of the tosses come up heads and five of them tails?

2. An integer–valued point in the xy–plane is a point (a, b) where both a and b are integers. How many integer–valued points are on or inside a circle of radius 4 centered at the origin?

3. Recall that $n!$ is the product of the first n positive integers, that is $2! = 2 \cdot 1$, $3! = 3 \cdot 2 \cdot 1$, and so on.

a. How many zeroes are at the end of 17!?

b. What is the smallest n such that $n!$ ends in exactly 37 zeroes?

4. a. Is 2003 a prime number?

b. What is the last digit of 2003^{2003}?

5. Suppose $f(X) = aX^2 + bX + c$ is a quadratic polynomial and a, b, c are rational numbers. Suppose that $f(n)$ is an integer whenever n is an integer. Are a, b, and c necessarily integers? If not, give an example.

6. John's father asks him to rake leaves and, in order to refine John's math skills, he offers John two possible choices of payment for his work:

A. one cent for the first bag, two cents for the second bag, four cents for the third bag, and so on: in other words 2^{n-1} cents for the n^{th} bag of leaves.

B. one dollar for the first bag, 2 dollars for the second bag, 3 dollars for the third bag, and so on: in other words n dollars for the n^{th} bag of leaves.

How many bags must John rake before option **A** becomes more profitable than option **B**?

7. a. What is the smallest number of coins (pennies, nickels, dimes, quarters, and half dollars) with which you can pay out any amount from 1 cent to 99 cents?

b. Suppose you can introduce coins of *any* denomination you wish. What is the smallest number of coins necessary to be able to pay out any amount from one cent to 99 cents?

8. Farmer Brown has eight logs, each of length 10 feet. What is the *maximum* area which he can enclose with the logs? For example, he could make a rectangle of height 10 feet and width 30 feet or a square with each side having length 20 feet. In the first case, he has enclosed 300 square feet and in the second case 400 square feet. The second choice is of course better than the first but what is the *largest* area which Farmer Brown can enclose?